

A fuzzy logic approach to reliability in archaeological virtual reconstruction

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Abstract.

In this paper, a new approach to a numeric definition of reliability for archaeological reconstruction models is proposed, basing on fuzzy logic. This approach allows to introduce for this important concept a numerical value based on verifiable elements, making possible a scientific evaluations of such reconstructions, After characterizing the process of building a reconstruction model from archaeological data and discussing the principal features of this reliability definition, the paper examines in detail the evaluation of reliability in a simple example. Suggestions for further research, including hints for a graphical representation of reliability in computer models and desirable options in user's interface, conclude the paper.

Keywords:

Fuzzy logic, archaeological reconstruction, reliability

1. Introduction

Virtual reconstruction of archaeological objects (artifacts, structures or archaeological sites) are often viewed by end-users as “objective truth”, leaving no space for analysis from an archaeological point of view, critics limiting to observations on artistic or computer graphics aspects. Issues such as reliability of the reconstruction, i.e. how accurately it returns the archaeological interpretation, are often obscured by the complexity (from a computer graphics point of view) of the virtual model. Moreover, since most models are mainly designed for public representation, and thus not subjected to scientific criticism, aspects such as accuracy of the reconstruction (from an archaeological point of view), reliability of the sources used for the reconstruction (photos, plans, drawings, historical sources, etc...) or relationship between the archaeological reality (how much was preserved) and the virtual reality (how much was virtually reconstructed) in many cases have no visual representation incorporated into the virtual model. Making use of information from various sources, each with different reliability characteristics and hidden uncertainties, the final result, the virtual reconstruction, will in most cases lack a pre-set standard of quality of information and thus restricting any critical analysis. Moreover, the end-user is left with the impression of completeness and singularity of the virtual reconstruction, whereas in many cases diverse and partial models will be available for display.

In recent years, the attention of researchers working on computer graphics models for archaeological reconstruction was therefore called on issues concerning their credibility and reliability.

It has been noted^{1, 2, 3} that research efforts have focused more on the optimization of technological aspects, (better-looking models while using less computer resources), than on problems arising from the availability of reliable reconstructions, from an archaeological scientific point of view. It has been emphasized that it is necessary to adopt a philological approach and to incorporate into the final model annotations and representation of alternate solutions,

along with presenting the difference between what is certain, what is reasonably deduced and what is simply a guess.

As far as we know, however, there is as yet no attempt at describing a ‘scientific’ procedure to evaluate such reliability, the term ‘scientific’ referring to, as Galileo first intended, what can be repeated with the same result (beyond experimental errors) by any other scientist. In a wider sense, it can be accepted as ‘scientific’ also what is based on someone’s authority, as far as it is clearly stated: it is more ‘scientific’ to state “I believe this reconstruction is valid because I am an expert in this field” than simply presenting the model without any further notice.

A complementary feature is a numerical measure of the reliability, quantifying the credibility of the above quoted statement: “...and I believe it is true at level x ”, x representing the degree of confidence the scientist believes in his or her guess, measured on a pre-determined scale of reliability. This degree of reliability is ultimately a *subjective* value, in the sense of De Finetti’s⁴ or Savage’s⁵ subjective approach to uncertainty. However, these subjective values need to be given a credible and objective nature, by referring them to computations that lead to their evaluation, substituting the expert’s statement with a chain of reasoning based on simpler facts and deductions.

The aim of this paper is to highlight the importance of expressing the reliability of the data used for the reconstruction (expressed as the level of confidence we have in our data), not only textually but, if possible, numerically (calculating an index of reliability, for example one that goes from archaeological reality – this is what we found – to pure imagination – that is what we think there was, but we have no proof whatsoever) and visually, either incorporated, or attached to the virtual reconstruction. Consequently, we are introducing an approach that proved to be very fruitful in archaeological research^{6,7,8,9} aimed at giving the reliability problem a scientific status: measurability and verifiability. It is suggested to apply concepts of fuzzy logic and fuzzy operations during the process of

reconstruction of the virtual model, the reliability index being estimated by applying concepts driven from the fuzzy set theory. Thus, the virtual model will be subject to a critical evaluation and analysis, the user will be aware of the reliability of the reconstruction and will be possibly able to decide the amount of information to be visualized, according to a chosen threshold of reliability.

2. Reliability in virtual reconstruction

The starting point of any virtual model is an archaeological or historical “reality”: the remains unearthed during excavation or a historical text describing the object to be virtually reconstructed. In the case of an archaeological object, several aspects may unbalance the accuracy of the future model: if the subject is an architectonic object, in many cases what is uncovered by archaeologists are only the foundations, these being completed by the modeler analyzing sparse material, using ethnographic parallels, textual descriptions or comparisons with better preserved sites (if they exist), the rest being completed by the “common sense” of the researcher based on his/her accumulated knowledge or, ultimately, imagination. The available material for the modeler are thus maps (sometimes in the traditional paper format, which need to be translated into a computer form), photos (that need to be adjusted with photogrammetry programs) and drawings in various formats. In the case of a reconstruction based on a textual (historic) description, a critical reading of the source is needed, then the model has to be confronted with architectural and physical laws.

Thus, we can see that from its starting point, the data upon which the model is built accumulate an unknown, and thus unpredictable and unquantifiable, degree of uncertainty and reliability. Therefore, without a degree of confidence, expressed by the reliability of the incorporated data, the final model cannot be subject to criticism from an archaeological point of view. Moreover, a non professional user of the model may be easily induced in error by the wholeness of the model and its apparent inviolability.

Let us analyze the process of creating an archaeological (reconstruction) model. It may be imagined that there is a “construction” set of parts that need to be assembled in order to generate the desired model. They may be referred to as a library of computer files, to be assembled by means of an appropriate software, or they may be existing in the archaeologist’s mind and be put together with pencil and paper. Each part consists of a geometry and a material (possibly composite) usually represented by means of a surface texture. By assembling these parts we are defining a new (partial) model formed by the union of the preceding ones, in a determined mutual position. The process ends when the partial model satisfies our needs, being close enough to the one we have in mind.

Consequently, the creation of an archaeological model is a stepwise process in which one starts from an initial model M_0 , possibly empty, placed at position x_0 ; at step n a new model M_{n+1} is built from M_n adding a new detail m_{n+1} in an absolute position x_{n+1} . Positions x are vectors containing all relevant information to put objects in place, uniquely determining their position in space. Relative position can be easily calculated from these, or vice versa the absolute position of an object may be easily determined by its relative position with reference to a fixed one.

An example may clarify the above statements.

In order to reconstruct a house, one starts from some graphical representation of the archaeological remains as unearthed, whose equivalent forms the initial model M_0 . Following, corresponding walls are added, taken from a “library” of walls and choosing what seem to be the closest to our mental representation of the final model (it should be pointed out that this choice is one among several available alternatives, others being rejected by the researcher on considerations based on his/her *a priori* knowledge). Their attributes include variables such as height, width, material and construction techniques. The first wall to be added is a new detail m_1 to be added “on top” of a foundation, positioned at x_1 . After adding all the necessary walls we obtain a new model M_k . The next step is to add features to these walls (are there openings in the walls?, are there windows? façade details? etc.) and eventually to place the roof if no other floors have to be reconstructed.

Every passage (adding a new feature) increases the completeness of the model and makes it more explanatory and pleasant to see. At the same time it reduces its reliability, which was almost total at the beginning being based on actual remains, since it implies the choice of newly added parts among many possible choices, a potentially risky operation. It is also clear that the resulting reliability depends on the mutual position of details: e. g. putting a credible roof at the bottom of a building would make the results totally unrealistic. Our analysis will try to compute this reliability reduction.

3. Computing the reliability of a virtual model

In order to compute a reliability index, it is necessary to establish a scale for reliability, which we propose to be the interval $[0, 1]$: zero reliability means “totally unreliable”, 1 means “absolutely reliable”.

Let us now consider the universe U of models M referring to a given archaeological reconstruction problem. This reference is necessary, because a model is not reliable *per se*, but only when referred to a specific problem. In this preliminary paper, we are not going to consider uncertainty factors due to temporal duration. In

other words we will assume that the archaeological model is temporally well-defined. This is a very special case of usual reconstruction problems: most human artifacts have been used and re-used for a long period of time, often varying in shape, structure and destination during their life. The reconstruction should refer to a specific phase of the artifact's life, but generally it is difficult if not impossible to precise what pertains to that phase and what does not. Thus the reconstruction problem is normally intrinsically ill-posed, adding temporal uncertainty (when?) to those related to position (where?) and modality (how?). As stated above, for the sake of simplicity we will presently ignore such a time uncertainty factor, which will be duly taken into account in forthcoming work.

We define as *reliability index of a model* a function $r: U \rightarrow [0, 1]$, attaching to every model $M \in U$ its reliability $r_M = r(M)$, that is a non-negative number less or equal than 1. A vector space X of positions will be considered as well.

Within U , we moreover define an operation A of aggregation consisting in "putting together" the two operands M_1 and M_2 in determined positions x_1 and x_2 . The result will be a new model M_3 in position x_3 : so, in more formal terms, the aggregation is an operation A on $U \times X$, that is a function on $U \times X \times U \times X$:

$$A((M_1, x_1), (M_2, x_2)) = (M_3, x_3).$$

While x_3 is useful to continue the aggregation process, it has no influence on the reliability of M_3 : any rigid movement of a model does not affect its reliability. It is in fact the relative position of details that counts. We may hence use the alternate notation:

$$A(M_1, M_2, q) = M_3$$

where q is a parameter taking into account the mutual position of M_1 and M_2 .

In this way the process of producing an archaeological reconstruction is modeled as a sequence of models M_0, M_1, M_2, \dots , each obtained from the previous one aggregating additional details to it in a determined position:

$$M_{k+1} = A(M_k, m_k, q_k)$$

where m_k denotes the details added in the last step and q_k the parameters resuming the mutual position of M_k and m_k .

In the following paragraphs we are going to discuss possible definitions of r and the possibility of computing the reliability of the result of an aggregation given those of the two operands.

4. A probabilistic model for reliability

An almost obvious approach to estimate the uncertainty of a virtual model is to apply a probabilistic perspective. Within this framework, it is necessary to define a probability P on U associating to every $M \in U$ a number $P(M)$ in such a way that:

$$\sum_{M \in U} P(M) = 1$$

and to define the reliability of a model M as a non-decreasing function of its probability $P(M)$, $P(M)$ itself as first choice.

Considering the sequence of models M_0, M_1, M_2, \dots that leads to the construction of a final model M , their probabilities are correlated each other by the following equation:

$$P(M_{k+1}) = P(M_k) P(M_{k+1} | M_k)$$

where the bar denotes, as usual, conditional probability, and takes into account both the compatibility of added details with the previous model and their position with regard to it. Therefore, iterating the previous formula, the probability and hence the reliability of M_{n+1} will be given by:

$$P(M_{n+1}) = p_0 p_1 p_2 \dots p_n$$

$$p_0 = P(M_0), p_k = P(M_{k+1} | M_k)$$

Let us now make some considerations on p_k 's, the conditional probabilities, which take into account both the compatibility and relative positioning of each added detail to the previous partial reconstruction. It can be easily verified that even if all such values are relatively as high as 0.8 (which is not always the case in archaeological reconstructions), after only ten passages the probability of the resulting model is as low as 0.1; in other words, after adding ten details, each with a 80% of compatibility with the nature and the position of all the previous ones, the resulting model is 10% reliable. Or, still worse, a model built up by aggregating a hundred details, each 95% compatible with the others ($p_k = 0.95$), is totally unreliable, its probability being 0.006.

Of course, the above does not show that archaeological modelers have to give up because their efforts would be unsuccessful even in the ideal conditions sketched previously. It simply suggests that a probabilistic approach leads to nowhere because of the normalization property of probability, which is the basis for the multiplicative law we were forced to adopt. In other words, probability is very unreliable as a measure of reliability.

5. The fuzzy logic approach

Fuzzy logic is a branch of mathematics based on fuzzy set theory. The latter, first proposed by Zadeh¹⁰ (1965), introduces special sets, called *fuzzy sets*, having a characteristic function that may vary between 0 and 1 and not only assume the two extreme values as for ordinary sets. Accordingly, we may define a *fuzzy truth function* f varying between 0 (false) and 1 (true) assuming also intermediate values (uncertain). It differs from probability as far as it needs no normalization: alternate statements do not need to have truth values adding up to 1. On this basic concepts a full-fledged theory has been constructed.^{11, 12}

Fuzzy logic has found applications in many fields of science. Being somehow related to AI, it fell in disgrace among theoretic computer scientists with it, and continued to be used principally in practical engineering industrial applications, where the main problem is defuzzification. We are going to use, on the contrary, only the fuzzy logical apparatus, with no defuzzification, in order to take into account the complexity of the archaeological word where a definitive "true" or "false" will be possible only after the invention of Well's time machine – that is, probably, never. In our opinion, in fact, archaeological "reality" is intrinsically fuzzy and as such it must be treated.⁹

In this way we may consider the reliability function as a fuzzy truth value of models.

For the logical operator F_AND corresponding to the logical AND, a "good" definition as discussed in Niccolucci *et al*⁹ uses the minimum of the fuzzy truth function f of the operands:

$$f(A \text{ F_AND } B) = \min(f(A), f(B)).$$

We may try now to define the reliability of a model resulting from the aggregation of intermediate models. We must split the problem of reliability of added details in two parts. The first one, absolute reliability $r^{(a)}$, takes into account the reliability of the object *per se*; the second, relative reliability $r^{(r)}$, takes into account the compatibility of the object with the context, that is with previously chosen details and the general characteristics of the model. For instance, it makes little sense to add architectural details related to warfare as crenellation, machicolation or arrow loops to the model of a medieval church (with the exception of fortress churches), so their relative reliability would be very low or some evidence should be given for their past existence. It is also necessary to take into account the reliability of the relative position, as shown by the roof example quoted above; this positional component of reliability will be denoted by $r^{(p)}$, meaning that it depends on the newly added detail with respect to the already created model.

Consequently, we can define:

$$r(M_{k+1}) = \min(r(M_k), r^{(a)}(m_k), r^{(r)}(m_k), r^{(p)}(q_k))$$

We see that this definition, applied to the previously considered examples to evaluate the probabilistic approach, would maintain as reliability of the final model the lowest value corresponding to the details added during the construction: in the two numerical cases, at least 80% in the first one and at least 95% in the second, a much more reasonable conclusion than the previous one.

The reliability of the final model M may be computed step by step, as a consequence of adding new details, or at the end, applying recursively the previous formula. Since the min function is associative one obtains in this case:

$$r(M) = \min_{k=1, \dots, n}(r_0, r_k^{(a)}, r_k^{(r)}, r_k^{(p)})$$

where r_0 is the reliability of the initial model and the reliability of each added detail is split into its absolute, relative and position components. In other words, the overall reliability of a model equals the worst one of its parts (according to content, compatibility or position).

It must be noticed that the above implies that the final reliability may depend on the order in which details are added, as this may influence the values of relative reliability $r^{(r)}$ and positional reliability $r^{(p)}$.

The above formula fits very well into a computational schema, and it is easy to compute during the model construction, as absolute reliability may be stored in the component library while relative and position reliability may be evaluated when the detail is added to the model.

6. Numerical evaluation of the reliability

Several methods are available to compute the numerical value of reliability. The simplest one, always available, is "Ask an expert". This may be implemented, for instance, in a model construction computer tool with an "aggregate" option, giving access to a library of details, each stored with its absolute reliability, and asking the operator for the values of the other two components when placing the detail in the model. Having disaggregated the construction of the model into sequential steps, each one involves such an evaluation, which is simpler to assign and may be better verified.

A more sophisticated approach may involve statistical analysis of variants and assign likelihood accordingly. Other methods may apply when purely geometrical considerations are involved. For instance, let us consider a circular temple. In this case a column of

radius R_1 is more likely to be placed in some positions than in others, e.g. closer to the exterior of the roof, and its position is determined by one coordinate, the distance x from the centre. Its position reliability may be then computed using a function as the following:

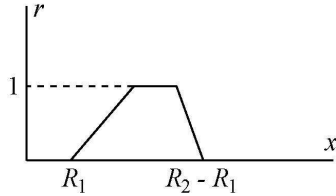


Figure 1: A typical piecewise-linear reliability function

decreasing when the pillar is too close to the border or to the centre, R_2 being the radius of the temple roof.

It must be evidenced that one method or another do not make the difference as long as they are documented. A reviewer (as every scholar may wish to be) will be able to evaluate single steps and agree or disagree with the reliability value assigned by the model creator, a task made easier by the decomposition of the overall model into more manageable elements. When many alternate solutions are available, they may be ranked in a reliability order and offered to the user, a visitor of an archaeological site or the scientist studying the work of a colleague.

7. Computing reliability: an example

In this paragraph we are going to compute the reliability of the reconstruction of a medieval belfry dating from the 12th century. The example aims at presenting the potential of the above definition, at clarifying some aspects and at showing that in practice computing the reliability is a complementary aspect of the documentation of the research work.

For the sake of clarity we are going to apply the above method to a very simple case, where only four steps are required to reach the final model. To have at hand a manageable case study, we are going to virtually demolish part of the (fortunately) still standing beautiful bell tower of the Cathedral of Spoleto, Italy. Let us fictitiously assume that in the past an earthquake (very frequent in that region) caused the collapse of the top of the tower, so remains include only a part of it below the roof and the bell cell, whose existence is uncertain.

We are not going to include here neither details on the interior of the tower nor on its building material. The corresponding initial model is M_0 , deriving from direct inspection of the remains (in fact, for this fictitious example it is our initial assumption), having therefore $r_0 = 1$.

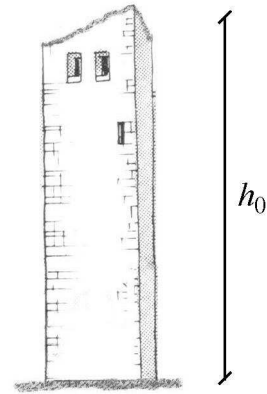


Figure 2: M_0 , the model based on the remains of the bell tower after the virtual earthquake

We decide that collapsed parts include the completion of the tower, probably the bell cell with its windows and the roof. Therefore, the first part to add is the completion m_1 of the tower, of an unknown height z_1 , while other dimensions (and the building material) need to match the existing remains. Aggregating this additional part we obtain the second partial model M_1 .

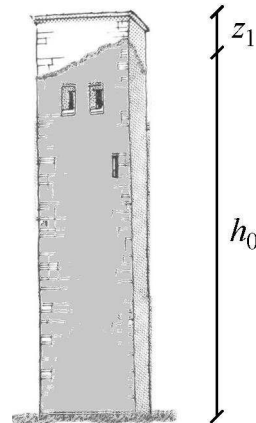


Figure 3: M_1 , the complete tower

Let us now compute the reliability of m_1 and hence of M_1 . Apart from the uncertainty concerning the height z_1 , there are no other issues about the part to add, such as shape or building material. So both relative reliability (compatibility) and the positional reliability (position) equal 1 for the chosen part, while the absolute reliability depends on z_1 . By a careful analysis of the collapsed material and measuring the tower foundations, an upper limit Z_1 and a lower limit Z_0 (or in any case $Z_0 = 0$) can be determined for z_1 , so $Z_0 < z_1 < Z_1$. In this interval central values will be more likely so we can use for the reliability of m_1 the values given by the following function:

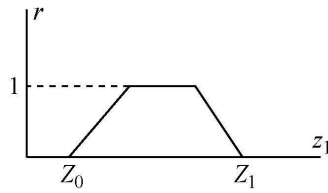


Figure 4: The reliability function for z_1

A different value of the overall reliability of M_1 will correspond to any choice of z_1 . We may decide to go for one of the most reliable ones, obtaining a total height of $h_1 = h_0 + z_1$.

The next step is to add the bell cell with its windows. First of all we must decide if there was such a cell; let us say there was indeed one and give to this assumption a reliability of 0.9 obtained by comparison with similar constructions. In this case several decisions must be taken:

- the height z_2 of the bell cell, basing on the already chosen h_1 , on similar buildings and on the collapsed material, obtaining as before a range $B_0 < z_2 < B_1$;
- the number n_2 of openings of the window(s), that is choose a double or triple window; let us assume $r(n_2) = 0.9$ for $n_2 = 2$, $r(n_2) = 0.6$ for $n_2 = 3$;
- the proportion s_2 of the window height to the cell height, based on similar buildings and on the collapsed material, with a range $S_0 < s_2 < S_1$.

For the range functions we may use a shape similar to the previous ones:

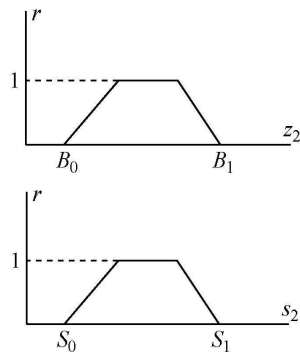


Figure 5: The reliability functions for z_2 and s_2

After choosing suitable values for all the parameters we obtain a model M_2 as the one shown in Figure 6. Note that even if one chooses the most feasible value for z_2 and s_2 , i.e. those giving $r = 1$, the reliability of the added part is 0.9 for the uncertainty concerning the window shape.

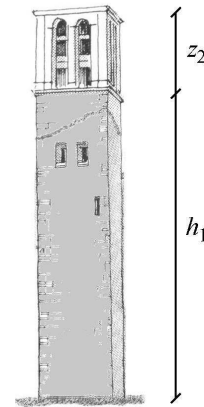


Figure 6: M_2 , the tower with the bell cell. Double windows have been chosen ($r = 0.9$)

Before adding the roof, it is possible that there is another small vertical part m_2 – let us assume that $r(\text{“YES”}) = 0.8$, $r(\text{“NO”}) = 0.7$ – of unknown height z_3 with the usual reliability function.

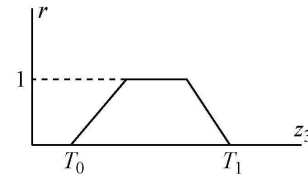


Figure 7: The reliability functions for z_3

The following figure shows a possible result for M_3 ; its reliability is 0.8 if z_3 is suitably chosen.

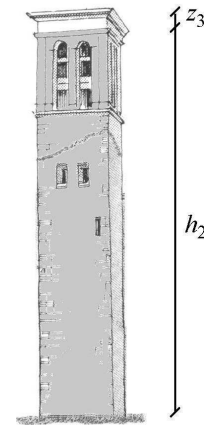


Figure 8: A possible result for M_3 ($r = 0.8$)

Note that in the figure cornices have been added to mark different architectural components. These are quite likely to appear, according to similar buildings, but possibly we ignore their exact shape. If we are confident in this at level at least 0.9, what we assume we are, this addition does not alter the overall reliability. This shows

another feature of the process we propose, simplifying it for very minor details in which we are rather confident.

We eventually arrive at the roof. There are several possibilities for this. One is a flat roof, which could possibly include crenellations, suggesting a defensive use of the tower, hinted also by the massive lower part with no windows. Otherwise a pyramidal roof is possible, with a quadrangular or an octagonal pyramid. It is interesting to note that both choices are equally valid and could possibly exist in different periods. This is one of the cases in which circumscribing precisely the temporal scope of the reconstruction becomes essential. A defensive purpose could only make sense in earlier times when the city might be subject to attacks. Later, the region become relatively calm, the church was in the centre of the town and the belfry could play no defensive role. In this case, the features of the reconstruction are determined by context in a very broad sense. Let us consider, however, the later phase (no defensive use) and assume the belfry had a normal roof, giving however this choice a somehow lower reliability, say $r(\text{"pyramid"}) = 0.9$. The pyramidal roof has two more parameters affecting reliability:

- the number n_4 of sides, say $r(n_4) = 0.8$ for $n_4 = 4$, $r(n_4) = 0.8$ for $n_4 = 6$ (no preference);
- the height z_4 of the roof, say with a range $R_0 < z_4 < R_1$ and with the usual reliability function (not shown).

In conclusion we obtain the model M_4 shown below.

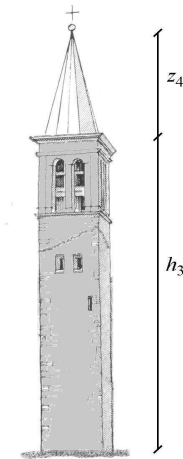


Figure 9: A possible final result M_4 ($r = 0.8$)

The reconstruction might lead to a completely different result. For instance, we may assume that the tower ended as M_1 and try to evaluate the reliability of

the resulting model M' . Here the dependence of r to time becomes essential, and we may decide after considering similar buildings from the same period that this choice has a reliability of 0.7 if referred to the first part of the 12th century. Here crenellation is very likely ($r = 0.9$)

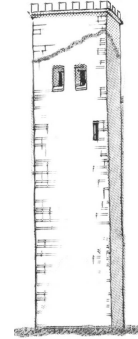


Figure 10: An alternate model M' ($r = 0.7$), featuring a shorter tower and merlons on top

Note that M' is the result of a branch after M_1 . The complete model production process may be thus represented as in the following diagram.

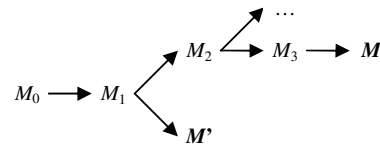


Figure 11: Diagram representing the model stepwise production process. Boldface represents final models

In fact the production of alternate models can be described as branches from the main path leading to the finally accepted model. For practical reasons, not all the possible paths are usually explored, but they could be in the future or by somebody else. The latter consideration shows another advantage of our approach: other researchers (or the same people in later work) may benefit from previous work without re-doing everything from scratch. They can just restart from an intermediate model, altering some detail in subsequent aggregations. All the previous work, and its reliability, is preserved, as a consequence of the modular aggregation process we are proposing.

8. Conclusions

Virtual models of archaeological objects are often shown as “closed objects”, leaving the viewer no possibility of critical analysis. Moreover, the complexity of the data used for the virtual reconstruction, originating from various sources, each with its own uncertainty degree, may influence in various ways the shape of the final model. One of the primary criteria for a model to be scientifically accepted is its data

transparency: the presentation of metadata and the data confidence level are thus a “must” step in transforming the virtual model from a “piece of art” into a scientific tool, subject to objective criticism also from an archaeological point of view. The importance of presenting the model together with its reliability is therefore of substantial importance.

In our opinion, fuzzy logic may be successfully applied to the evaluation of the degree of confidence of virtual archaeological reconstruction models, as it has been shown here by a simple and artificial example and will be shown in future work on real case studies. With this approach, the user is provided with a numerical index, estimating where the model is placed between archaeological reality (what has been found) and pure modeler’s imagination (i.e. what he/she believes there was, with no proof whatsoever). Moreover, the display system could be designed in order to allow the user to choose to visualize only models passing a reliability threshold, to have the possibility to visualize the various levels of uncertainty, corresponding to alternative models. In GIS applications, for instance, various techniques have been proposed to represent uncertain data¹³: one method, uses color hue and/or saturation to display different degrees of reliability; another shows uncertain components as “ghosts”. When visualizing an archaeological virtual reconstruction, the user interface might for instance provide a linear meter for reliability or even allow for a slider, to be used to regulate the desired (or accepted) reliability level. Other devices might allow to visualize alternate solutions.

Incorporating absolute reliability into model parts as an attribute is a simple task when using an XML-compliant description of the archaeological reconstruction model, as proposed by one of the authors in a previous paper.¹⁵ To represent graphically the reliability when using this approach, it may be converted into some feature enabled in the viewer. We are going to examine these aspects in a forthcoming paper on the reliability of different reconstructions of the legendary Porsenna’s mausoleum in Chiusi, Italy, dating since the 16th century to present days.

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